Roll No. 661301

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MT/ D-13

STOCHASTIC METHODS

Paper –M.Tec.–1.4

Time allowed : 3 hours] *[Maximum marks : 60*

Note : *Attempt **five** questions in all, selecting at least one question from each of the four units.*

Unit–I

1. (a) If a three-digit decimal number is chosen at random, find the probability that exactly k digits are greater than or equal to 5, for $0 \le k \le 3$. 4

(b) What is Bayes' rule ? Discuss its applications. 4

(c) What do you understand by discrete-random vectors ? Explain in brief. 4

2. (a) Show that density of the sum of the two nonnegative independent random variables is the convolution of the individual densities. 5

(b) Define the expectation E [X] of a random variable X and explain its significance. 2

(c) Given a random variable X and two functions h(x) and g(x) satisfying the conditions $h(x) \le g(x)$ for all x, show that

$E[h(X)] \le E[g(X)]$, whenever both expectations exist. 5

Unit–II

3. (a) Define and explain Stochastic Process with illustration. 2

(b) Briefly explain the Elementary Renewal Theorem. 3

(c) For a server with Poisson job arrival stream at an average rate of 60 per hour, determine the probability that the time interval between successive job arrivals is

(i) longer than 4 minutes

(ii) shorter than 8 minutes

(iii) between 2 and 6 minutes. 7

4. (a) Derive the mathematical expression for steady-state availability. Clearly state assumptions, if any. 6

(b) Assuming respective steady-state availabilities of 0.99, 0.999, 0.9999, 0.99999, and 0.999999, determine the limiting expected downtime in minutes, for an interval of duration one year. 6

Unit–III

5. (a) Define the n-step transition probability and show that the n-step transition matrix may be obtained by multiplying the matrix P by itself n-times. Symbols used carry usual meanings. 6

(b) Show that if state i is recurrent and state i communicates with state j, then state j is recurrent. 6

6. (a) Discuss the mean time spent in transient states. 6

(b) Prove that an ergodic birth-death process is time reversible. 6

Unit–IV

7. (a) How queues are described ? Explain with an example. Also show a typical network of queues. 4

(b) Explain the following :

(i) open queuing network

(ii) closed queuing network 4

(c) What is confidence of intervals ? Explain the steps involved in obtaining a confidence interval for the parameter θ from a random sample X1, X2, ...Xn. 4

8. (a) For an arithmetic unit of a computer system with a modulo-m online fault detector, as the modulus m varies, the average detection latency y also varies. For the data given in Table–1, with two observations of y for each value of m, determine the parameters a and b by performing a least squares fit of the curve $y = am^b$. 8

Table-1

m_i	y_i (μs)	
3	1.45	1.50
5	1.30	1.26
7	1.21	1.23
11	1.10	1.08
13	1.05	1.03

(b) Write a short note on non-linear regression. 4